

Exercise 42

For the following exercises, find $(f \circ g)$ and the domain for $(f \circ g)(x)$ for each pair of functions.

$$f(x) = \frac{1}{x^2 - 1}, \quad g(x) = \sqrt{x + 1}$$

Solution

Calculate $(f \circ g)(x)$ by plugging the formula for $g(x)$ in where x is in the formula for $f(x)$.

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= \frac{1}{(\sqrt{x + 1})^2 - 1} \\ &= \frac{1}{(x + 1) - 1} \\ &= \frac{1}{x}\end{aligned}$$

The denominator of this rational function cannot be zero at any step, and the square root of a negative number cannot be taken.

$$x + 1 \geq 0 \quad \text{and} \quad x \neq 0$$

Solve for x .

$$x \geq -1 \quad \text{and} \quad x \neq 0$$

Therefore, the domain of $(f \circ g)(x)$ is

$$[-1, 0) \cup (0, \infty).$$